Performance Analysis of CSMA/CA (Carrier Sense Multiple Access with Collision Avoidance) Protocols for Wireless Local Area Networks in Multipath and Shadowing Environments

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Abstract

The channel throughput and packet delay of wireless media access control (MAC) protocols with Rayleigh fading, shadowing and capture effect are analyzed. We consider CSMA/CA protocols as the wireless MAC protocols, since CSMA/CA protocols are based on the standard for wireless Local Area Networks (LANs) IEEE 802.11. We analyze the channel throughput and packet delay for three types of CSMA/CA protocols; Basic CSMA/CA, Stop-and-Wait CSMA/CA and 4-Way Handshake CSMA/CA. We calculate the capture probability of an AP (Access Point) in the channel with Rayleigh fading, shadowing and near-far effects, and derive the throughput and packet delay for the protocols. We have found that the performance of CSMA/CA in radio channel model is reduced 50 percent more than those of error free channel model in low traffic load, while the throughput and packet delay of CSMA/CA in radio channel show better performance than those of error free channel model in high traffic load. We also found that the 4-Way Handshake CSMA/CA protocol is superior to the other CSMA/CA protocols in high traffic load.

I Introduction

Many interesting developments have taken place in wireless communications and portable communications in recent years. More and more stations connect to wireless LANs and demand for various wireless services, which support data, voice and moving pictures, has rapidly increased. The costs for installation and relocation for cable LAN have increased. However, wireless LANs offers many advantages in installation, maintenance and relocation from the points of view of cost and efficiency. Wireless LAN manufacturers currently offer a number of nonstandardized products based on conventional radio modem technology, spread-spectrum technology in ISM (Industrial, Scientific and Medical) bands, and infrared technology [1].

Since 1990, IEEE Project 802.11 committee has worked to establish a universal standard for wireless LANs protocol for interoperability between competing products [2], and ETSI (European Telecommunications Standards Institute) set up an ad hoc group to investigate radio LANs in 1991 [3]. One of the important research issues in wireless LANs is the design and analysis of Medium Access Control (MAC) protocols. In this paper, we consider a Carrier Sense Multiple Access/Collision Avoidance (CSMA/CA) protocol, which is a basic mechanism of the IEEE 802.11 MAC protocol, and analyze the performance of CSMA/CA protocols by using a mathematical method based on a renewal theory.

MAC protocols for wireless communications have been widely studied. There are some analytical studies for CSMA/CA protocols [4],[5], and some simulation studies [6],[7]. However, Chen assumes that CSMA/CA is a non-persistent CSMA in [4], and Chhaya calculates the throughput of CSMA/CA with a simple model in [5]. Other studies do not present analytical approaches. There are also many studies for ALOHA family protocols in fading channel and shadowing [8]-[11]. However, the characteristics of the CSMA/CA cannot be described by the ALOHA protocols and have not yet been analyzed in fading channel model.

In this paper, we present an exactly analytical approach for the channel throughput and the normalized packet delay of CSMA/CA protocols in Rayleigh fading, log-normal shadowing and near-far effect. We consider the centralized wireless LAN configuration, and focus on the performance of Access Point (AP) in wireless LAN. We analyze the performances of three type of CSMA/CA protocols and compare the throughput and normalized packet delay with each others.

This paper is organized as follows. In Section I, the propagation model and system model for CSMA/CA protocols is described. The throughput of three types of CSMA/CA is analyzed in Section III, and in Section IV, packet delay is calculated. In Section V, some numerical results are reported. Finally, we give concluding remarks in Section VI.

II System Descriptions

II-1. Propagation Model

We focus on the performance of AP in the infra-structure wireless networks. We consider that AP is located in the center of the infra-structure configuration and the other terminals are distributed in the Basic Service Area (BSA) with the given spatial distribution density function. The radio
channel can be characterized statistically by three independent, multiplicative, propagation mechanisms, namely multi-
path fading, shadowing and groundwave propagation [12].
The groundwave propagation gives rise to the near-far ef-
effect and determines the area-mean power \( w_a \), which means the received power averaged over some area. Therefore, the
normalized area-mean power received from a wireless ter-

dinal at a distance \( r_i \) from the access point is taken to have the form

\[ w_a = r_i^{-\xi}, \quad (1) \]

with the exponent, \( \xi \), typically takes values in the rage of three to four. We assumed that shadowing is superimposed
on the near-far effect. This fluctuation is described by a
lognormal distribution of the local-mean power \( w_a \) about
the area-mean power \( w_a \) with logarithmic standard deviation \( \sigma_s \). We also assume that power control is not used
and that Rayleigh fading is an accurate characterization of
the link fading process. While Rician fading is also of
interest, it is a much more difficult type of fading process
to treat analytically. See [12] and [13] for a more detailed in-
door propagation model. Thus, the instantaneous received
power \( w_0 \) of a signal from a wireless terminal is exponen-
tially distributed about the local-mean power \( w_L \). Taking
into account Rayleigh fading, log-normal shadowing and
near-far effects, the unconditional probability density func-
tion(pdf) of the instantaneous power \( w_0 \) of a received packet is [9],[12]

\[
f_{w_0}(w_0) = \int_0^\infty \int_0^\infty \frac{1}{w_L} \exp\left(-\frac{w_0}{w_L}\right) \frac{f(r_i)}{\sqrt{2\sigma_s w_L}} \exp\left(-\frac{\ln^2(r_i^2/w_L)}{2\sigma_s}\right) \, dr_i \, dw_L, \quad (2)
\]

where \( f(r_i) \) is the pdf of the propagation distance describ-
ing the spatial distribution. We consider the uniform spatial
distribution in which we assume that the wireless terminal
are uniformly distributed over a circle of unit radius about
the access point. In this case, the pdf of the propagation
distance is given by \( f(r_i) = 2r_i \), \( r_i \in (0, 1) \) [12], [14].

II-2. Power Capture Model

The test packet can be received successfully, that is, it cap-
tures the receiver in the presence of other overlapping or
interfering packets, if its instantaneous power is larger than
the instantaneous joint interference power by at least certain
threshold factor \( z \). This effect is the capture effect, and the
threshold factor is called as capture ratio [15],[16]. As is
often done in the literature, the instantaneous power is as-
sumed to remain approximately constant for the time inter-
val of packet duration. To use a convenient method of an-
alyzing capture probabilities, we consider the weight func-
tion approach for the Rayleigh fading channels [9] based on
Laplace transforms [17].

We consider that a wireless network consists of \( M \) ter-

dinals \( \{s_1, s_2, \ldots, s_i, \ldots, s_M\} \). We define \( R_n \) as the set of
terminals, which means \( n \) terminals transmit a packet at the
same time. The \( s_i \) denotes a receiver and wants to receive
the packet from a certain transmitter \( s_i \). If the \( s_i \) receives the packet successfully from \( s_i \), the instantaneous
signal power \( w_s \) should exceed the joint interference signal
power \( w_L \) from \( n-1 \) terminals \( \{R_n - \{s_i\}\} \) by the capture
ratio \( z \). However, the \( s_i \) does not receive the packet suc-
cessfully from \( s_i \) if only \( w_s \) is captured from \( w_L \), since the
\( w_s \) also includes the joint interference signal with the mul-

tipath fading, shadowing and near-far effect by own itself.
Let \( w_f \) be the joint interference signal for only \( s_i \) and \( w_0 \) denote the desired signal power of a packet, then \( w_0 \) and \( w_f \) are inclued in the \( w_s \).

In order to find the capture probability, denotes \( q(n|z) \),
for \( n \) colliding packet, we first consider that the \( q(s_1|z) \) de-
notes the probability of capture for a packet from the \( s_i \) with
distance of \( r_i \). Given the local mean power \( w_f \), it can be expressed as

\[
q(s_1|z) = \int_0^\infty \int_0^\infty \frac{f(r_1)}{\sqrt{2\sigma_s w_f}} \exp\left(-\frac{\ln^2(r_1^2/w_f)}{2\sigma_s}\right) \cdot \left\{ \phi_{w_f}(\frac{z}{w_f}) \right\} \, dr_1 \, dw_L. \quad (3)
\]

Here, \( \phi(\cdot) \) is the one side Laplace image of the pdf of the
instantaneous joint interference power \( w_f \), defined as

\[
\phi_{w_f}(s) = \int_0^\infty \exp(-sx) f(w_f)(x) \, dx. \quad (4)
\]

Using (2), \( \phi(\cdot) \) can be expressed as

\[
\phi_{w_0}(s) = \int_0^\infty \int_0^\infty \frac{1}{1+sw_L} \frac{f(r)}{\sqrt{2\sigma_s w_L}} \exp\left(-\frac{\ln^2(r_i^2/w_L)}{2\sigma_s}\right) \, dr \, dw_L. \quad (5)
\]

Further, if the interference received power \( w_L \) is due to
coincident accumulation of \( n \) independent fading signals for \( n \)
wireless terminals, the joint pdf of the received power is the
\( n \)-fold convolution of the pdf of the individual signal power
[8], [15]. Thus, the probability of capture, given that \( n \) ter-

minals transmit packets at the same time, can be expressed as

\[
q(n|z) = \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty f(r_1) f(s_1|r_1, z) \cdot \phi_{w_f}(\frac{z}{w_f}) f(S_2|z) \cdot \left\{ \phi_{w_L}(\frac{z}{w_L}) \right\}^{n-1} \, dw_1 \, dw_2 \, dr_1 \, dr_2, \quad (6)
\]

where

\[
\begin{align*}
f(s_1|r_1, z) &\triangleq \frac{1}{\sqrt{2\sigma_s w_0}} \exp\left(-\frac{\ln^2(r_i^2/w_0)}{2\sigma_s}\right) \\
f(S_2|r_2, z) &\triangleq \frac{1}{\sqrt{2\sigma_s w_0}} \exp\left(-\frac{\ln^2(r_i^2/w_0)}{2\sigma_s}\right) \\
f(r_1) &= \begin{cases} 2r_1 & \text{if } 0 < r_1 < 1 \\ 0 & \text{otherwise} \end{cases}
\end{align*}
\]
\[ f(r_2) = \begin{cases} \frac{2r_2}{\pi} & \text{if } 0 < r_2 < 1 \\ 0 & \text{otherwise} \end{cases} \]  

(7)

where, the \(f(s)\) is the pdf of the random distance of a transmitting terminal from the receiving terminal and this is assumed by uniform spatial distribution. Note that \(f(s_1 | r_1, z)\) and \(f(s_n | r_2, z)\) are statistically independent. Finally, the capture probability, conditional on the number of \(n\) interferers, is the three-fold integral form as

\[
q(n; z) = \frac{2}{\pi} \int_{-\infty}^{\infty} r_1 \exp(-x_1^2) \left[ \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-y_1^2) \right] dx_1 dr_1 \\
- \frac{2}{\pi} \int_{-\infty}^{\infty} r_2 \exp(-x_2^2) \left[ \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-y_2^2) \right] dx_2 dr_2,
\]

(8)

where

\[
f(x_1, y_1) = \sqrt{2 \pi r_1} \exp \left\{ \frac{-1}{2 \sigma_y^2} (y_1 - x_1) \right\} \cdot \frac{1}{\sqrt{\pi}} \exp \left\{ \frac{-1}{2 \sigma_x^2} (x_1 - y_1) \right\}
\]

\[
f(x_2, y_2) = \sqrt{2 \pi r_2} \exp \left\{ \frac{-1}{2 \sigma_y^2} (y_2 - x_2) \right\} \cdot \frac{1}{\sqrt{\pi}} \exp \left\{ \frac{-1}{2 \sigma_x^2} (x_2 - y_2) \right\}
\]

(9)

The \(q(n; z)\) can be obtained using the Hermite polynomial methods [17]. The probability \(q_n\) that one out of \(n\) packets captures the access point is found from

\[ q_n = nq(n; z) \]  

(10)

### II-3. CSMA/CA Protocols and System Model

The IEEE 802.11 MAC protocol supports coexisting asynchronous and time-bounded services using different priority levels with different Inter Frame Space (IFS) delay controls. In wireless communication environments, packet transmission suffers from “hidden terminal”, so IEEE 802.11 MAC protocol provides three alternative ways of packet transmission flow control [2]. In this paper, we consider three types of CSMA/CA according to the packet transmission flow control. First, actual data packet is used only for packet transmission which is called Basic CSMA/CA. Second, immediate positive acknowledgements are employed to confirm the successful reception of each packet. We call this scheme Stop-and-Wait (SW) CSMA/CA. The last is 4-Way Handshake (4-WH) CSMA/CA which use Request To Send (RTS) and Clear To Send (CTS) packets prior to the transmission of the actual data packet.

In the CSMA/CA, we assume that the time is slotted with a slot size \(a\) (propagation delay/packet transmission time), and all terminals are synchronized to start transmission only at slot boundaries. To analyze better exact throughput of the CSMA/CA, we use a finite population (\(M\) terminals). To use the advantage of the memoryless property [18], we assume that each terminal has periods, which are independent and geometrically distributed, in which there are no packets. We consider only the case of statistically identical terminals. A terminal generates a new packet with probability \(g\) (0 < \(g\) < 1). We consider that \(g\) includes new arrival and rescheduled packets during a slot. If a terminal has no packet to transmit, we call this terminal an empty terminal and we call the opposite a ready terminal. We assume that each ready station starts packet transmission with probability \(p\) (0 < \(p\) < 1) and this \(p\) is related to CW in the backoff delay. The duration of the packet transmission period is assumed to be fixed as a unit of time 1, so the packet transmission time is composed of \(1 + (1/a)\) slots.

In this paper, we consider the CSMA/CA as a hybrid protocol of the slotted 1-persistent CSMA and \(p\)-persistent CSMA. We assume that a channel state consists of a sequence of regeneration cycles composed of idle and busy periods. An idle period (denoted by \(I\)) is the time in which the channel is idle and no terminal attempts to access the channel. A busy period (denoted by \(B\)) occurs when one or more terminals attempt to transmit packets, and ends if no packets
have been accumulated at the end of the transmission. Let $U$ be the time spent in useful transmission during a regeneration cycle and $S$ be the channel throughput. The throughput $S$ can be obtained by the above three terms, and the normalized packet delay is also calculated using throughput.

### III Throughput Analysis

#### III-1. Basic CSMA/CA

In the following, we consider the Basic CSMA/CA protocol, and calculate the expectation of the idle period, the busy period and the useful transmission period. The throughput of CSMA/CA is then derived. In CSMA/CA, channel states are illustrated as in Fig. 1(a). Let us introduce some notations which define channel states. In Fig. 1(a), the busy period is divided into several sub-busy periods such that the $j$th sub-busy period, which is denoted by $B^{(j)}$, is composed of a transmission delay (denoted by $D^{(j)}$) and transmission time (denoted by $T^{(j)}$).

In the sub-busy period $B^{(1)}$, $D^{(1)}$ is DIFS delay. However, $D^{(j)}$ is the stochastic random variable, if $j \geq 2$. $B^{(j)}$ is composed of a DIFS delay, $D^{(j)}$ and $T^{(j)}$. The DIFS delay is assumed to have $l$ slots, and the size of DIFS is $l = t \times x$.

In the case of the Basic CSMA/CA model, transmission period $T^{(j)}$ is fixed at $1 + a$, whether the transmission is successful or not. Let $J$ be the number of sub-busy periods in a busy period. The busy period $B$ and the useful transmission period $U$ are simply given by

$$B = \sum_{j=1}^{J} B^{(j)}, \quad U = \sum_{j=1}^{J} U^{(j)}. \tag{11}$$

Next, we have to find the number of sub-busy periods in a busy period. In CSMA/CA every terminal transmits a pending packet after it detects the free medium for greater than or equal to a DIFS. Therefore, the busy period continues in the case that a packet is generated during the last transmission period as well as the last DIFS delay. Let $TP$ be the sum of the last transmission period and the last DIFS delay, then $TP$ is $1 + a + f$ in the Basic CSMA/CA model. Since $J$ is geometrically distributed, the distribution and the expectation of $J$ are

$$\Pr[J = j] = \left[1 - (1 - g)^{(TP/a)M}\right]^{j-1} \cdot (1 - g)^{(TP/a)M}, \quad \mathbb{E}[J] = \frac{1}{1 - (1 - g)^{(TP/a)M}} \quad j = 1, 2, \ldots. \tag{12}$$

The $B^{(1)}$ occurs when one or more packets arrive in the last slot of the idle period, and $B^{(2)}$ occurs when one or more packets arrive in $T^{(1)}$. Since the length of $B^{(j)} (j \geq 3)$ is independent of that of $B^{(2)}$ and identically distributed, the expectation of $B^{(j)} (j \geq 2)$ is $(\mathbb{I} - 1) \times E[B^{(2)}]$. In the same manner, we get the expectation of $U^{(j)}$ easily. Thus, the expectation of busy period and useful transmission time is given by

$$\mathbb{E}[B] = E[B^{(1)}] + (J - 1) E[B^{(2)}], \quad U = E[U^{(1)}] + (J - 1) E[U^{(2)}] \tag{13}$$

Since the idle period is geometrically distributed, distribution and expectation of the duration for an idle period is given by

$$\Pr[I = ka] = (1 - g)^{M(k-1)} \cdot (1 - (1 - g)^{M}), \quad \mathbb{E}[I] = \frac{\alpha}{1 - (1 - g)^{M}}, \quad k = 1, 2, \ldots. \tag{14}$$

To find $E[D^{(j)}]$ and $E[U^{(j)}]$, let $P_{n}(X)$ be the probability that $n$ packets arrive in $M$ users during $X$ slots, given $n \geq 1$. The $P_{n}(X)$ is expressed as

$$P_{n}(X) = \left(\frac{M}{n}\right)^{[1-(1-g)^{X/a}]^{n}(1-g)^{XM-n/X}} \frac{1}{1-(1-g)^{XM/a}} ; \quad n = 1, 2, \ldots M. \tag{15}$$

Furthermore, let $N^{(j)}_{0}$ be the number of packets accumulated at the end of the transmission period, then the distribution of $N^{(j)}_{0}$ is expressed as

$$\Pr[N^{(j)}_{0} = n] = P_{n}(TP) \quad j = 2, 3, \ldots \tag{16}$$

In order to find the distribution of $D^{(j)}$ when $N^{(j)}_{0} = n$ and $j \geq 2$, we consider $k$ to be the number of slot boundaries as $k = 0, 1, 2, \ldots, D^{(j)}$ is greater than equal to $k$ slots in the following cases; $n$ terminals, which are already scheduled to transmit a packet, do not transmit a packet and $(M-n)$ empty terminals generate no packet during $k$ slots. Thus, we have

$$\Pr[D^{(j)} \geq kn] = \Pr[N^{(j)}_{0} = n] = (1-p)^{kn}(1-g)^{k(M-n)}. \tag{17}$$

We can derive the expectation of $D^{(j)}$, given $N^{(j)}_{0} = n$, unconditioning on $N^{(j)}_{0}$ in (17), the expectation of $D^{(j)} (j \geq 2)$ can be calculated.

$$E[D^{(j)}] = \left\{ \begin{array}{c} f[1 - (1 - g)^{M}] \quad ; \quad j = 1 \\ \frac{1}{1-(1-g)^{TP/aM}} \left( \sum_{k=1}^{\infty} (1 - p)^{k} - (1-g)^{(TP/a)[(1-p)^{k} - (1-g)^{k}]^{M}} \right) \quad ; \quad j = 2, 3, \ldots \end{array} \tag{18} \right.$$  

Using (13), (14) and (18), we obtain the sum of expectations of the busy period and the idle period as

$$\mathbb{E}[B + I] = f[1 - (1 - g)^{M}] + 1 + a + \frac{1}{(1-g)^{(TP/a)M}} \left( (f + 1 + a)[1 - (1 - g)^{(TP/a)M}] + \alpha \sum_{k=1}^{\infty} (1 - p)^{k} - (1-g)^{(TP/a)[(1-p)^{k} - (1-g)^{k}]^{M}} \right) - \alpha(1-g)^{(TP/a)M} \sum_{k=1}^{\infty} (1 - g)^{kM} \cdot \frac{\alpha}{1 - (1 - g)^{M}} \tag{19}$$
We calculate the expected value of useful transmission time $E[U^{(j)}]$. In order to calculate $E[U^{(j)}]$, we consider the condition when $N_0(j) = n$ and $D^{(j)} \geq ka$. Then, we have

$$E[U^{(j)}|D^{(j)} \geq ka, N_0^{(j)} = n]$$

$$= \begin{cases} 
\sum_{i=1}^{n-1} \binom{M}{n} p^i (1 - p)^{n-i} g(i - 1 | z) & ; k = 0 \\
\sum_{i=1}^{n-1} \binom{M}{n} p^i (1 - p)^{n-i} g^l (1 - g)^{M - n - l} (i + l) q(i + l | z) & ; k > 0 \\
\cdot \sum_{i=1}^{n-1} \binom{M - n}{l} l! (1 - g)^{M - n - l} (i + l) q(i + l | z) & ; k > 0 \\
\cdot \sum_{i=1}^{n-1} \binom{M}{n} p^i (1 - p)^{n-i} g(i - 1 | z) & ; k > 0 \\
\end{cases}$$

(20)

where $i$ means the number of terminals which have transmit the packet with probability $p$ in the $n$ backlogged terminals, and $l$ means the number of terminals which generate the new packet with probability $g$. Using conditional expectation in (20), we can obtain the mean successful transmission period.

Since $U^{(1)}$ is the useful transmission time when one or more packets arrive during the last slot of the previous idle period, it is equal to $P_1(1)$ in (15). Thus, we have

$$U = E[U^{(1)}] + (J - 1) E[U^{(2)}]$$

$$= \left(1 - \frac{1}{1 - (1 - g)^M} g(i - 1 | z) + \frac{1}{(1 - g)^{TP/o} M} \right)$$

$$\cdot \sum_{n=1}^{M} \left\{ \binom{n}{i} p^i (1 - p)^{n-i} g^l (1 - g)^{M - n - l} (i + l) q(i + l | z) \right\}$$

$$\cdot \sum_{n=1}^{M} \left\{ \binom{M - n}{l} l! (1 - g)^{M - n - l} (i + l) q(i + l | z) \right\}$$

$$\cdot \sum_{n=1}^{M} \left\{ \binom{M}{n} [1 - (1 - g)^{TP/o} n(1 - g)^{TP/o} (M - i) / (1 - g)^{TP/o} M] \right\}$$

(21)

Dividing (19) from (21), we obtain the throughput of a slotted Basic CSMA/CA system composed of $M$ identical users, each user has geometric arrival rate $g$, slot time $a$ and DIFS delay $f$.

III.2. Stop-and-Wait CSMA/CA

In the following, we consider the SW CSMA/CA protocol and calculate the throughput of SW CSMA/CA. For SW CSMA/CA, channel states are illustrated in Fig. 1(b), in which $\beta$ means the normalized time of SIFS and $\delta$ does that of ACK packet. Here the parameters and assumptions are the same as in the case of Basic CSMA/CA except that successful transmission period $(TP_S)$ is given by $1 + \beta + \delta + 2a + f$, when the transmission is successful. Note that $TP_S$ includes the DIFS delay since packets, generated in the period of the last DIFS delay, have to wait for the channel goes idle. When the packet transmission is unsuccessful, the ACK packet transmission period is omitted and the unsuccessful transmission period $(TP_F)$ is $1 + a + f$. Denoting by $TP$ the duration of the $j$th transmission period in the busy period, then $(j + 1)$th transmission period depends only on $TP$. This is why the success of $(j + 1)$th transmission is determined by the number of arrivals during the $j$th transmission period. Hence, given a transmission period $(TP)$, the length of the remainder of the busy period is a function of $TP$, and its average period is denoted by $B(TP)$. Similarly the average useful transmission period in the remainder of the busy period is denoted by $U(TP)$.

$$B(TP) = d(TP) + \left\{ (TP_S + [1 - (1 - g)^{TP/o} a] B(TP_S) \right\} u(TP)$$

$$+ \left\{ TP_F + [1 - (1 - g)^{TP/o} a] B(TP_F) \right\} [1 - u(TP)]$$

$$U(TP) = \left\{ 1 + [1 - (1 - g)^{TP/o} a] U(TP_S) \right\} u(TP)$$

$$+ \left\{ 1 - (1 - g)^{TP/o} a \right\} [1 - u(TP)] (22)$$

where,

$$d(1) = f[1 - (1 - g)^M]$$

$$d(TP) = \frac{\alpha}{1 - (1 - g)^{TP/o} M}$$

$$\cdot \sum_{k=1}^{\infty} \left\{ (1 - p)^k - (1 - g)^{TP/o} \right\}$$

$$\cdot \left\{ (1 - p)^k - (1 - g)^k \right\}^M$$

$$\cdot (1 - g)^{TP/o} M \sum_{k=1}^{\infty} (1 - g)^{kM}$$

(23)

$$u(1) = \left(1 - \frac{1}{1 - (1 - g)^M} g(i - 1 | z) \right)$$

$$u(TP) = \sum_{n=1}^{M} \left\{ \binom{n}{i} p^i (1 - p)^{n-i} g^l (1 - g)^{M - n - l} (i + l) q(i + l | z) \right\}$$

$$\cdot \sum_{n=1}^{M} \left\{ \binom{M - n}{l} l! (1 - g)^{M - n - l} (i + l) q(i + l | z) \right\}$$

$$\cdot \sum_{n=1}^{M} \left\{ \binom{M}{n} [1 - (1 - g)^{TP/o} n(1 - g)^{TP/o} (M - i) / (1 - g)^{TP/o} M] \right\}$$

where $d(TP)$ and $u(TP)$ are derived from (18) and (20), respectively. If $j \geq 2$, we have to consider that $TP$ is the
case of both $TP_S$ and $TP_F$. Since a busy period is induced by the first slot before it starts, we get
\[
\overline{T} = B(1) ; \overline{U} = U(1) \quad (24)
\]

Since the duration of successful transmission is different from that of unsuccessful transmission, $B(TP_S)$, $B(TP_F)$, $U(TP_S)$ and $U(TP_F)$ are calculated respectively. Substituting $TP$ by $TP_S$ and $TP_F$ in (22), we can obtain two equations with two unknowns $B(TP_S)$ and $B(TP_F)$ which can be solved easily. The average length of idle period is the same as in (14). Thus, we find the throughput of SW CSMA/CA protocol.

\[
S = \frac{U(1)}{B(1) + \frac{\lambda}{[1-(1-\gamma)^2]}a} \quad (25)
\]

### III-3. 4-Way Handshake CSMA/CA

We now proceed to calculate the throughput of the 4-Way Handshake CSMA/CA. Since packet transmission is not absolutely reliable in wireless communication environments, IEEE 802.11 provides 4-Way handshaking with a CSMA/CA mechanism. The carrier sense mechanism is achieved by distributing medium busy reservation information through an exchange of special small RTS and CTS frames prior to the actual data frame. If a collision occurs during the RTS packet transmission period, the packet transmission is terminated immediately and a new packet transmission is started.

The channel model for slotted 4-WH CSMA/CA is shown in Fig. 1(c). If the RTS packet transmission is successful, transmission period $(T^{(J)})$ is composed of RTS packet transmission period $(\gamma)$, CTS packet transmission period $(\alpha)$, data packet transmission period $(\delta)$, ACK packet transmission period $(1)$, SIFS (3$\beta$) and 4 propagation delay $(4a)$. We denote that $TP_{1S}$ is the sum of the successful transmission period and DIFS delay. Therefore, $TP_{1S}$ is $1 + \gamma + \alpha + \delta + 3\beta + 4a + f$. In the unsuccessful case, $T^{(J)}$ is the sum of RTS packet transmission period and an SIFS. Let $TP_{1F}$ be the sum of the last unsuccessful transmission period and DIFS, then $TP_{1F}$ is $\gamma + a + f$.

In order to calculate the throughput of 4-WH CSMA/CA, we modify the analysis on previous Section III-2. Substituting $TP_S$ and $TP_F$ with $TP_{1S}$ and $TP_{1F}$ respectively, we can easily obtain $B(TP_{1S})$ and $U(TP_{1F})$. Using (22), (24) and (25) and calculating recursive forms of $B(TP_{1S})(U(TP_{1S}))$ and $B(TP_{1F})(U(TP_{1F}))$, we can obtain $B(1)$ and $U(1)$. Then, we can derive the throughput of 4-WH CSMA/CA.

### IV Delay Analysis

#### IV-1. Basic CSMA/CA

In a packet transmission network, the performance is usually represented by channel throughput and packet delay. We denote the expected packet delay $L$ to be the average time from when a packet is generated to when it is successfully received.

In order to calculate the packet delay, we use offered traffic $(G)$ and throughput $(S)$. We use the average number of retransmission for a packet which is $(G/S - 1)$. We now introduce the average delay $R$ for a packet from the sensing channel to the accessing channel. This is one of the following three cases: (1) A packet arrives and senses the channel as the idle period $(I)$. (2) A packet arrives and senses the channel as the delay period $(D)$. (3) A packet arrives and senses the channel as the transmission period $(B - D)$. In the case of 1), an arbitrary packet has arrived and will find the channel idle with probability $\overline{I}/(\overline{I} + \overline{T})$. The average delay is DIFS. In the case of 2), a packet has arrived and will find the channel in the delay period with probability $\overline{D}/(\overline{I} + \overline{T})$. In this case, the average delay is also DIFS. In the last case, a packet has arrived and will find the channel in the period of another packet transmission period with probability $(\overline{T} - \overline{D})/(\overline{I} + \overline{T})$. In this case, the packet waits for the channel to be idle and delays by backoff algorithm. The average delay can be calculated by residual life period in renewal theory [18]. So we can get the average delay $\overline{R}$ as

\[
\overline{R} = \overline{T}/(\overline{I} + \overline{T}) + \overline{D}/(\overline{I} + \overline{T}) + \overline{D}/(\overline{I} + \overline{T}) \times \frac{[TP + E[D^{(2)}]]^2}{2[TP + E[D^{(2)}]]} \quad (26)
\]

In (26), we can obtain $E[D^{(2)}]$ using (18) and calculate by

\[
\overline{D} = E[D^{(1)}] + (\overline{I} - 1)E[D^{(2)}] \quad (27)
\]

Let $T$ be the packet transmission period and $\overline{T}$ is $(1 + \alpha)$ in the Basic CSMA/CA model. We can obtain the normalized average packet delay by

\[
L = \left( \frac{G}{S} - 1 \right) [T + \overline{T} + \overline{R}] + T + \overline{R} \quad (28)
\]

where $\overline{R}$ denotes random delay for a collided packet that waits for $\overline{T}$ before sensing the channel.

#### IV-2. Stop-and-Wait ARQ CSMA/CA

As in the case of Basic CSMA/CA, we calculate the average delay for the interval of successive transmission by

\[
\overline{R} = \overline{T}/(\overline{I} + \overline{T}) + \overline{D}/(\overline{I} + \overline{T}) + \overline{D}/(\overline{I} + \overline{T}) \times \left\{ P_{\text{Succ}} \frac{[TP_S + d(TP_S)]^2}{2[TP_S + d(TP_S)]} \right\} \quad (29)
\]

where $TP_S$ is the sum of the last successful transmission period and DIFS with $1 + \beta + \gamma + 2a + f$ and $TP_F$ is the sum of the last unsuccessful transmission period and the DIFS with $1 + a + f$. $P_{\text{Succ}}$ denotes the probability of a successful packet transmission which is $(G/S)$ and $P_{\text{Fail}}$ is $1 - P_{\text{Succ}}$. Other notations are the same as those of previous
Section IV-1., but $\overline{D}$ has to be calculated differently. $\overline{D}$ can be obtained by $D(1)$ as follows

\[ D(1) = f + \left\{ d(TP_S) + \left[ 1 - (1 - g)^{TP_S/a} \right] D(TP_S) \right\} u(1) \]
\[ + \left\{ d(TP_f) + \left[ 1 - (1 - g)^{TP_f/a} \right] D(TP_f) \right\} \cdot [1 - u(1)] \]  

(30)

where $d(TP_S)$ and $d(TP_f)$ can be obtained, substituting $TP$ with $TP_S$ and $TP_f$ in (22). $D(TP_S)$ and $D(TP_f)$ can be calculated by substituting $1$ with $TP_S$ and $TP_f$ respectively in (30). Since the backoff delay is determined by the previous transmission period, we have to calculate the backoff delay in both the cases of successful and unsuccessful transmission period. Let $T_S$ be $1 + \beta + \gamma + 2a$ and $T_F$ be $1 + a$. Then, the normalized delay $L$ in SW CSMA/CA is obtained easily by substituting former $T$ with $T_F$ and later $T$ with $T_S$ in (28).

**IV-3. 4-Way Handshake CSMA/CA**

In 4-WH CSMA/CA, the packet transmission period is different to that of SW CSMA/CA. Since we have assumed that $TP_{4S}$ is $1 + \gamma + \theta + \delta + 3\beta + 4a + f$, $TP_{4F}$ is $\gamma + a + f$. $T_{4S}$ is $1 + \gamma + \theta + \delta + 3\beta + 4a$ and $T_{4F}$ is $\gamma + a$, we calculate the average delay for the interval of successive transmission ($\overline{T}$) by

\[ \overline{T} = \frac{T}{B + 1} f + \frac{\overline{D}}{B + 1} f + \frac{\overline{T} - \overline{D}}{B + 1} \cdot \begin{cases} P_{Succ} \left[ \frac{(TP_{4S} + d(TP_{4S}))^2}{2[TP_{4S} + d(TP_{4S})]} \right] \\ + P_{fail} \left[ \frac{(TP_{4F} + d(TP_{4F}))^2}{2[TP_{4F} + d(TP_{4F})]} \right] \end{cases} \]  

(31)

where $P_{Succ}$ denotes the probability that a packet transmission is successful which is $\left( G/S \right)$ and $P_{fail}$ is $1 - P_{Succ}$ as the same in the Section IV-2, has to be calculated in a manner similar to that of SW CSMA/CA. $\overline{T}$ is a recursive form as in (30) by substituting $TP$ with $TP_{4S}$ and $TP_{4F}$ with $TP_{4F}$. Then, the normalized delay $L$ in 4-WH CSMA/CA is easily obtained easily by substituting former $T$ with $T_{4F}$ and later $T$ with $T_{4S}$ in (28).

**V Numerical Results**

Based on the analysis presented in the previous sections, some numerical results are shown and the performances of three types of CSMA/CA are compared in this section. Fig. 2 plots the capture probability for the number of terminals, when the capture ratio $z$ is varied. The capture probabilities

**Figure 2:** Capture probability for the number of colliding terminals ($\sigma_z = 6dB, \xi = 4$)

**Figure 3:** Throughput and packet delay vs. offered load of Basic CSMA/CA for varying of the number of terminals ($\alpha = 0.01, p = 0.03, f = 0.06, \overline{f} = 0.06, \sigma_z = 6dB, \xi = 4, z = 4$) (a) Throughput vs. offered load (b) Normalized packet delay vs. offered load

}\end{itemize}
Figure 4: Throughput and packet delay vs. transmission probability $p$ of SW CSMA/CA for varying of the offered load ($\alpha = 0.01, p = 0.03, f = 0.06, \delta = 0.06, \beta = 0.03, \gamma = 0.06, \sigma_z = 6dB, \xi = 4, z = 4, M = 50$) (a) Throughput vs. $p$ (b) Normalized packet delay vs. $p$.

are decreased exponentially when the number of colliding terminals are in the range of 1 to 10. However, if the number of colliding terminals is increased above 10, the capture probabilities are converged to a finite limit. It also can be seen that the capture probabilities are decreased exponentially as the capture ratio $z$ increases.

Fig. 3 shows the effect of the offered load $G$ on the throughput and the normalized delay for Basic CSMA/CA when the number of terminals varied. Note that as the number of terminals increased, the throughput is not decreased but saturated asymptotically in Fig. 3(a). In the case of Fig. 3(b), the normalized packet delay is decreased as the number of terminals is increased, while it is linearly increased as the offered load is increased.

To investigate the performance of SW ARQ CSMA/CA for varying of transmission probability $p$, the throughput and normalized packet delay are represented in Fig. 4. We note that the performance of SW CSMA/CA is not degraded as the increasing of transmission probability. In usual cases of $p$-persistent CSMA, the performance is degraded when the transmission probability is increased above a specific value [19],[20]. The performance of CSMA/CA in the error free channel model also shows similar drift for the variation of transmission probability $p$ [21]. The increase of the $p$ enhances the performance, due to the beneficial effect of power capture. According to the increase of the $p$, the probability of packet collision is increased, while the chance to capture a packet is increased. The reason why the capture probability is converged to a finite limit.

Fig. 5 reports throughput and packet delay versus capture ratio $z$ and offered load $G$. In the Fig. 5(a), we note that throughput is decreased more rapidly in low offered traffic while the throughput is less decreased in high offered load. This means that the power capture is more effective in high

Figure 5: Throughput and packet delay vs. capture ratio $z$ of 4-WH CSMA/CA for varying of the offered load ($\alpha = 0.01, p = 0.03, f = 0.06, \delta = 0.06, \beta = 0.03, \gamma = 0.06, \sigma_z = 6dB, \xi = 4, M = 20$) (a) Throughput vs. $z$ (b) Normalized packet delay vs. $z$. 

traffic loads. In the case of Fig. 5(b), the packet delay is increased linearly as the increase of the capture ratio $\alpha$.

The performance comparison of three types of CSMA/CA is represented in Fig. 6. Note that curves including polygons mean the analytical results in the error free channel model [21]. They can be obtained by substituting the capture ratio with 0 in the presented equations, when the number of colliding terminals is above 2. In the case of the error free channel model, The Basic CSMA/CA shows better performance than that of other two CSMA/CA protocols in low traffic load, while 4-WH CSMA/CA superior to others in high traffic load. In the case of fading, shadowing and power capture model, the performance of 4-WH CSMA/CA is always better than that of the other two protocols. Moreover, we note that the performance of CSMA/CA in the fading channel model is worse than that in the error free channel model when the traffic is low. This is the reason why the performance is sensitive to the multipath and the shadowing. However, the throughput of CSMA/CA in the fading channel model is increased continuously with the increase of the offered load, while the throughput in the error free channel model decreased when the traffic increase above specific point. This is due to the capture effect.

Finally, we note that 4-WH CSMA/CA protocol is more appropriate than Basic CSMA/CA or SW ARQ CSMA/CA in practical wireless communication environments.

VI Conclusions

In this paper, we have analyzed the performance of CSMA/CA protocols with power capture, operating on a channel impaired by Rayleigh fading, lognormal shadowing and near-far effect. We have considered three types of CSMA/CA protocols, including Basic, SW ARQ and 4-WH CSMA/CA, and have analyzed their throughput and packet delay.

To analyze the performance of CSMA/CA, we have considered capture probability in fading and shadowing channels. We have found that capture probability is converged to a finite limit as the number of colliding terminals is increased. Then, we have developed a new analytical approximation for the performance of CSMA/CA protocols with Rayleigh fade, lognormal shadowing and power capture effect. From the analytical results, we have found that the throughput of CSMA/CA protocol is not decreased as the increase of the number of terminals and the offered load. We have also found that the performance of CSMA/CA is enhanced as the increase of transmission probability $p$ and is sensitive the capture ratio $\alpha$. Extensive numerical results have been presented showing that 4-WH CSMA/CA protocol is a more attractive protocols than the other two types of CSMA/CA in practical wireless communication environments.

The main contributions of this paper are threefold: (1) the development of an analytical approach for evaluating the performance of CSMA/CA protocols in fading and shadowing channels (2) the performance comparison of three types of CSMA/CA protocols, and (3) the performance comparison of CSMA/CA in error free channel model and fading channel.

References


