

# Throughput and Packet Delay Analysis of CSMA/CA Protocols for Wireless LANs in Multipath Fading, Shadowing and Near-far Effects

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**Abstract** The channel throughput and packet delay of wireless media access control (MAC) protocols with Rayleigh fading, shadowing and capture effect are analyzed. We consider CSMA/CA protocols as the wireless MAC protocols, since CSMA/CA protocols are based on the standard for wireless Local Area Networks (LANs) IEEE 802.11. We analyze the channel throughput and packet delay for three types of CSMA/CA protocols; Basic CSMA/CA, CSMA/CA with Acknowledgement and 4-Way Handshake CSMA/CA. We calculate the capture probability of a receiver in the channel with Rayleigh fading, shadowing and near-far effects, and derive the throughput and packet delay for the protocols. We have found that the performance of CSMA/CA in radio channel model is reduced 50 percent more than those of error free channel model in low traffic load. We also found that the 4-Way Handshake CSMA/CA protocol is superior to the other CSMA/CA protocols.

## I. Introduction

Since 1990, IEEE Project 802.11 committee has worked to establish a universal standard for wireless LANs protocol for interoperability between competing products [1], and ETSI(European Telecommunications Standards Institute) set up an ad hoc group to investigate radio LANs in 1991 [2]. One of the important research issues in wireless LANs is the design and analysis of Medium Access Control (MAC) protocols. In this paper, we consider a Carrier Sense Multiple Access/Collision Avoidance (CSMA/CA) protocol, which is a basic mechanism of the IEEE 802.11 MAC protocol, and analyze the performance of CSMA/CA protocols by using a mathematical method based on a renewal theory.

MAC protocols for wireless communications have been widely studied. There are some analytical studies for CSMA/CA protocols [3], and some simulation studies [4]. However, Chen assumes that CSMA/CA is a non-persistent CSMA, and Chhaya calculates the throughput of CSMA/CA with a simple model. Other studies do not present analytical approaches. There are also many studies for ALOHA family protocols in fading channel and shadowing [5]. However, the characteristics of the CSMA/CA cannot be described by the ALOHA protocols and have not yet been analyzed in fading channel model.

In this paper, we present an exactly analytical approach for the channel throughput and the normalized packet delay of CSMA/CA protocols in Rayleigh fading, shadowing and near-far effect. We consider the centralized wireless LAN configuration, and focus on the performance of Access Point (AP) in wireless LAN.

## II. System Description

The radio channel can be characterized statistically by three independent, multiplicative, propagation mechanisms, namely multipath fading, shadowing and groundwave propagation [6]. The groundwave propagation gives rise to the near-far effect and determines the area-mean power  $w_a$ , which means the received power averaged over some area. Therefore, the normalized area-mean power received from a wireless terminal at a distance  $r_i$  from the access point is taken to have the form

$$w_a = r_i^{-\xi} \quad (1)$$

with the exponent,  $\xi$ , typically takes values in the range of three to four. We assumed that shadowing is superimposed on the near-far effect. This fluctuation is described by a lognormal distribution of the local-mean power  $w_L$  about the area-mean power  $w_a$  with logarithmic standard deviation  $\sigma_s$ . We also assume that power control is not used and that Rayleigh fading is an accurate characterization of the link fading process.

The instantaneous received power  $w_0$  of a signal from a wireless terminal is exponentially distributed about the local-mean power  $w_L$ . Taking into account Rayleigh fading, lognormal shadowing and near-far effects, the unconditional probability density function (pdf) of the instantaneous power  $w_0$  of a received packet is [5],[6]

$$f_{w_0}(w_0) = \int_0^\infty \int_0^\infty \frac{1}{w_L} \exp\left(-\frac{w_0}{w_L}\right) \frac{f(r_i)}{\sqrt{2}\sigma_s w_L} \cdot \exp\left\{-\frac{\ln^2(r_i^\xi w_L)}{2\sigma_s}\right\} dr_i dw_L \quad (2)$$

where,  $f(r_i)$  is the pdf of the propagation distance describing the spatial distribution. We consider the uniform spatial distribution in which we assume that the wireless terminal are uniformly distributed over a circle of unit radius about the access point. In this case, the pdf of the propagation distance is given by  $f(r_i) = 2r_i$ ,  $r_i \in (0, 1)$  [6].

Let  $w_f$  be the joint interference signal for only  $s_i$  and  $w_0$  denote the desired signal power of a packet, then  $w_0$  and  $w_f$  are included in the  $w_s$ .

In order to find the capture probability, denotes  $q(w_0|z)$ , with distance of  $r_i$ . Given the local mean power ( $w_L$ ), it can be expressed as

$$q(w_0|z) = \int_0^\infty \int_0^\infty \frac{f(r_i)}{\sqrt{2}\sigma_s w_f} \exp\left\{-\frac{\ln^2(r_i^\xi w_f)}{\sqrt{2}\sigma_s}\right\} \cdot \left\{\phi_{w_i}\left(\frac{z}{w_L}\right)\right\} dr_i dw_L \quad (3)$$

where

$$f(r_i) = \begin{cases} 2r_i & \text{if } 0 < r_i < 1 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Here,  $\phi(\ast)$  is the one side Laplace image of the pdf of the instantaneous joint interference power  $w_f, \xi = 4$ .

The exact calculation of  $q(w_0|z)$  can be obtained using the Hermite polynomial methods. In this paper, we consider three types of CSMA/CA according to the packet transmission flow control ; Basic CSMA/CA, Stop-and-Wait (SW) CSMA/CA and 4-Way Handshake (4-WH) CSMA/CA. In the CSMA/CA, we assume that the time is slotted with a slot size  $a$  (propagation delay/packet transmission time), and all terminals are synchronized to start transmission only at slot boundaries. To analyze better exact throughput of the CSMA/CA, we use a finite population ( $M$  terminals). A terminal generates a new packet with probability  $g$ . We consider that  $g$  includes new arrival and rescheduled packets during a slot. We assume that each ready station starts packet transmission with probability  $p$ . The duration of the packet transmission period is assumed to be fixed as a unit of time 1.

In this paper, we consider the CSMA/CA as a hybrid protocol of the slotted 1-persistent CSMA and  $p$ -persistent CSMA. We assume that a channel state consists of a sequence of regeneration cycles composed of idle and busy periods. An idle period (denoted by  $I$ ) is the time in which the channel is idle and no terminal attempts to access the channel. A busy period (denoted by  $B$ ) occurs when one or more terminals attempt to transmit packets, and ends if no packets have been accumulated at the end of the transmission. Let  $U$  be the time spent in useful transmission during a regeneration cycle and  $S$  be the channel throughput.

### III. Throughput Analysis

#### A. Basic CSMA/CA

In the following, we consider the Basic CSMA/CA protocol, and calculate the expectation of the idle period, the busy period and the useful transmission period. The throughput of CSMA/CA is then derived. We obtain the sum of expectations of the busy period and the idle period as in (5), the detail calculations are driven in [8].

$$\begin{aligned} \bar{B} + \bar{I} &= f[1 - (1 - g)^M] + 1 + a + \frac{1}{(1 - g)^{(TP/a)M}} \\ &\quad \left( (f + 1 + a)[1 - (1 - g)^{(TP/a)M}] \right. \\ &\quad + a \sum_{k=1}^{\infty} \left\{ (1 - p)^k - (1 - g)^{(TP/a)k} \right. \\ &\quad \cdot \left. \left. [(1 - p)^k - (1 - g)^k] \right\}^M \right. \\ &\quad \left. - a(1 - g)^{(TP/a)M} \sum_{k=1}^{\infty} (1 - g)^{kM} \right) \\ &\quad + \frac{a}{1 - (1 - g)^M} \end{aligned} \quad (5)$$

We calculate the expected value of useful transmission time  $E[U^{(j)}]$ . In order to calculate  $E[U^{(j)}]$ , we consider the condition when  $N_0(j) = n$  and  $D^{(j)} \geq ka$ . Then, we have

$$E[U^{(j)} | D^{(j)} \geq ka, N_0^{(j)} = n] = \begin{cases} np(1 - p)^{n-1}q(w_0|z) & ; k = 0 \\ \begin{cases} [np(1 - p)^{(n-1)}(1 - g)^{(M-n)} + (1 - p)^n \\ (M - n)g(1 - g)^{(M-n-1)}] q(w_0|z) \end{cases} & ; k > 0 \end{cases} \quad (6)$$

where  $i$  means the number of terminals which have transmit the packet with probability  $p$  in the  $n$  backlogged terminals, and  $l$  means the number of terminals which generate the new packet with probability  $g$ . Using conditional expectation in (6), we can obtain the mean successful transmission period. Since  $U^{(1)}$  is the useful transmission time when one or more packets arrive during the last slot of the previous idle period. Thus, we have

$$\bar{U} = E[U^{(1)}] + (\bar{J} - 1)E[U^{(2)}] \quad (7)$$

Dividing (5) from (7), we obtain the throughput of a slotted Basic CSMA/CA system composed of  $M$  identical users, each user has geometric arrival rate  $g$ , slot time  $a$  and DIFS delay  $f$ .

#### B. Stop-and-Wait CSMA/CA

In the following, we consider the SW CSMA/CA protocol and calculate the throughput of SW CSMA/CA. Let  $\beta$  be the normalized time of SIFS and  $\delta$  does that of ACK packet. Here the parameters and assumptions are the same as in the case of Basic CSMA/CA except that successful transmission period ( $TP_S$ ) is given by  $1 + \beta + \delta + 2a + f$ , when the transmission is successful. This is why the success of  $(j + 1)$ th transmission is determined by the number of arrivals during the  $j$ th transmission period. Hence, given a transmission period ( $TP$ ), the length of the remainder of the busy period is a function of  $TP$ , and its average period is denoted by  $B(TP)$ . Similarly the average useful transmission period in the remainder of the busy period is denoted by  $U(TP)$ .

$$\begin{aligned} B(TP) &= d(TP) \\ &+ \left\{ (TP_S + [1 - (1 - g)^{(TP_S/a)}] B(TP_S)) \right\} u(TP) \\ &+ \left\{ TP_F + [1 - (1 - g)^{(TP_F/a)}] B(TP_F) \right\} \\ &\cdot [1 - u(TP)] \end{aligned} \quad (8)$$

$$\begin{aligned} U(TP) &= \left\{ 1 + [1 - (1 - g)^{(TP_S/a)}] U(TP_S) \right\} u(TP) \\ &+ \left\{ [1 - (1 - g)^{(TP_F/a)}] U(TP_F) \right\} [1 - u(TP)] \end{aligned}$$

where  $d(TP)$  and  $u(TP)$  are derived from (5) and (6), respectively. If  $j \geq 2$ , we have to consider that  $TP$  is the case of both  $TP_S$  and  $TP_F$ . Since a busy period is induced by the first slot before it starts, we get

$$\bar{B} = B(1) ; \bar{U} = U(1) \quad (9)$$

Since the duration of successful transmission is different from that of unsuccessful transmission,  $B(TP_S)$ ,  $B(TP_F)$ ,  $U(TP_S)$  and  $U(TP_F)$  are calculated respectively. Substituting  $TP$  by  $TP_S$  and  $TP_F$  in (5), we can obtain two equations with two unknowns  $B(TP_S)$  and  $B(TP_F)$  which can be solved easily. The average length of idle period is the same as in (??). Thus, we find the throughput of SW ARQ CSMA/CA.

$$S = \frac{U(1)}{B(1) + \frac{a}{[1 - (1 - g)^M]}} \quad (10)$$

#### C. 4-Way Handshake CSMA/CA

We now proceed to calculate the throughput of the 4-Way Handshake CSMA/CA. Since packet transmission is not absolutely reliable in wireless communication environments, IEEE 802.11 provides 4-Way handshaking with a CSMA/CA

mechanism. The carrier sense mechanism is achieved by distributing medium busy reservation information through an exchange of special small RTS and CTS frame prior to the actual data frame.

We assume that normalized packet transmission of RTS and CTS are and respectively. The channel model for slotted 4-WH CSMA/CA is shown in Fig.1. If the RTS packet transmission is successful, transmission period ( $T^{(j)}$ ) is composed of RTS packet transmission period ( $\gamma$ ), CTS packet transmission period ( $\theta$ ), data packet transmission period (1), ACK packet transmission period ( $\delta$ ), 3 SIFS ( $3\beta$ ) and 4 propagation delay ( $4a$ ). We denote that  $TP_{4S}$  is the sum of the successful transmission period and DIFS delay. Therefore,  $TP_{4S}$  is  $1 + \gamma + \theta + \delta + 3\beta + 4a + f$ . In the unsuccessful case,  $T^{(j)}$  is the sum of RTS packet transmission period and an SIFS. Let  $TP_{4F}$  be the sum of the last unsuccessful transmission period and DIFS, then  $TP_{4F}$  is  $\gamma + a + f$ . In order to calculate the throughput of 4-WH CSMA/CA, we modify the analysis on previous Section. Substituting  $TP_S$  and  $TP_F$  with  $TP_{4S}$  and  $TP_{4F}$  respectively, we can easily obtain  $B(TP)$  and  $U(TP)$ . Using (8), (9) and (10) and calculating recursive forms of  $B(TP_{4S})(U(TP_{4S}))$  and  $B(TP_{4F})(U(TP_{4F}))$ , we can obtain  $B(1)$  and  $U(1)$ . Then, we can derive the throughput of 4-WH CSMA/CA.

## IV. Delay Analysis

### A. Basic CSMA/CA

In order to calculate the normalized packet delay ( $L$ ), we use offered traffic ( $G$ ) and throughput ( $S$ ). We use the average number of retransmission for a packet which is  $(G/S - 1)$ . We now introduce the average delay  $R$  for a packet from the sensing channel to the accessing channel. This is one of the following three cases : 1) A packet arrives and senses the channel as the idle period. 2) A packet arrives and senses the channel as the delay period ( $D$ ). 3) A packet arrives and senses the channel as the transmission period. In the case of 1), an arbitrary packet has arrived and will find the channel idle with probability  $\bar{I}/(\bar{I} + \bar{B})$ . The average delay is DIFS. In the case of 2), a packet has arrived and will find the channel in the delay period with probability  $\bar{D}/(\bar{I} + \bar{B})$ . In this case, the average delay is also DIFS. In the last case, a packet has arrived and will find the channel in the period of another packet transmission period with probability  $(\bar{B} - \bar{D})/(\bar{I} + \bar{B})$ . In this case, the packet waits for the channel to be idle and delays by backoff algorithm. The average delay can be calculated by residual life period in renewal theory. Let  $T$  be the packet transmission period and  $T$  is  $(1 + a)$  in the Basic CSMA/CA model. So we can get the average delay  $R$  as

$$\bar{R} = \frac{\bar{I}}{\bar{B} + \bar{I}}f + \frac{\bar{D}}{\bar{B} + \bar{I}}f + \frac{\bar{B} - \bar{D}}{\bar{B} + \bar{I}} \left[ \frac{(TP + E[D^{(2)}])^2}{2(TP + E[D^{(2)}])} \right] \quad (11)$$

In (11), we can obtain  $E[D^{(2)}]$  using (5) and calculate by

$$\bar{D} = E[D^{(1)}] + (\bar{J} - 1)E[D^{(2)}] \quad (12)$$

We can obtain the normalized average packet delay by

$$L = \left( \frac{G}{S} - 1 \right) [T + \bar{Y} + \bar{R}] + T + \bar{R} \quad (13)$$

where,  $Y$  denotes random delay for a collided packet that waits for  $Y$  before sensing the channel.

### B. Stop-and-Wait CSMA/CA

As in the case of Basic CSMA/CA, we calculate the average delay for the interval of successive transmission by

$$\bar{R} = \frac{\bar{I}}{\bar{B} + \bar{I}}f + \frac{\bar{D}}{\bar{B} + \bar{I}}f + \frac{\bar{B} - \bar{D}}{\bar{B} + \bar{I}} \cdot \left\{ P_{Succ} \left[ \frac{[TP_S + d(TP_S)]^2}{2[TP_S + d(TP_S)]} \right] + P_{Fail} \left[ \frac{[TP_F + d(TP_F)]^2}{2[TP_F + d(TP_F)]} \right] \right\}. \quad (14)$$

where,  $TP_S$  is the sum of the last successful transmission period and DIFS with  $1 + \beta + \gamma + 2a + f$  and  $TP_F$  is the sum of the last unsuccessful transmission period and DIFS with  $1 + a + f$ .  $T_S$  is the successful transmission period ( $1 + \beta + \gamma + 2a$ ) and  $T_F$  is the unsuccessful transmission period ( $1 + a$ ).  $P_{Succ}$  denotes the probability of a successful packet transmission which is  $(G/S)$  and  $P_{Fail}$  is  $1 - P_{Succ}$ . Other notations are the same as those of previous Section A., but  $\bar{D}$  has to be calculated differently.  $\bar{D}$  can be obtain by  $D(1)$  as follows

$$D(1) = f + \left\{ d(TP_S) + \left[ 1 - (1 - g)^{(TP_S/a)} \right] D(TP_S) \right\} u(1) + \left\{ d(TP_F) + \left[ 1 - (1 - g)^{(TP_F/a)} \right] D(TP_F) \right\} \cdot [1 - u(1)] \quad (15)$$

where  $d(TP_S)$  and  $d(TP_F)$  can be obtained using (5) and (6).  $D(TP_S)$  and  $D(TP_F)$  can be calculated by substituting 1 with  $TP_S$  and  $TP_F$  respectively. Since the backoff delay is determined by the previous transmission period, we have to calculate the backoff delay in both the cases of successful and unsuccessful transmission period. Then, normalized delay  $L$  in SW CSMA/CA is obtained easily by substituting former  $T$  by  $T_F$  and later  $T$  by  $T_S$  in (13).

### C. 4-Way Handshake CSMA/CA

In 4-WH CSMA/CA, the packet transmission period is different to that of SW CSMA/CA. Since we have assumed that  $TP_{4S}$  is  $1 + \gamma + \theta + \delta + 3\beta + 4a + f$ ,  $TP_{4F}$  is  $\gamma + a + f$ ,  $T_{4S}$  is  $1 + \gamma + \theta + \delta + 3\beta + 4a$  and  $T_{4F}$  is  $\gamma + a$ , we calculate the average delay for the interval of successive transmission ( $\bar{R}$ ) by

$$\bar{R} = \frac{\bar{I}}{\bar{B} + \bar{I}}f + \frac{\bar{D}}{\bar{B} + \bar{I}}f + \frac{\bar{B} - \bar{D}}{\bar{B} + \bar{I}} \cdot \left\{ P_{Succ} \left[ \frac{[TP_{4S} + d(TP_{4S})]^2}{2[TP_{4S} + d(TP_{4S})]} \right] + P_{Fail} \left[ \frac{[TP_{4F} + d(TP_{4F})]^2}{2[TP_{4F} + d(TP_{4F})]} \right] \right\} \quad (16)$$

$\bar{D}$  is a recursive form as in(15) by substituting  $TP_S$  with  $TP_{4S}$  and  $TP_F$  with  $TP_{4F}$ . Then, the normalized delay  $L$  in 4-WH CSMA/CA is easily obtained easily by substituting former  $T$  by  $T_{4F}$  and later  $T$  by  $T_{4S}$  in (13).

## V. Numerical Results

Fig. 1 shows the effect of the offered load  $G$  on the throughput and the normalized delay for Basic CSMA/CA when the

number of terminals varied. Note that as the number of terminals increased, the throughput is not decreased but saturated asymptotically in Fig. 1(a). In the case of Fig. 1(b), the normalized packet delay is decreased as the number of terminals is increased, while it is linearly increased as the offered load is increased. Fig. 2 reports throughput and packet delay versus capture ratio  $z$  and offered load  $G$ . In the Fig. 2(a), we note that throughput is decreased more rapidly in low offered traffic while the throughput is less decreased in high offered load. In the case of Fig. 2(b), the packet delay is increased linearly as the increase of the capture ratio  $z$ . The performance comparison of three types of CSMA/CA is represented in Fig. 3. Note that curves including polygons mean the analytical results in the error free channel model [8]. In the case of the error free channel model, The Basic CSMA/CA shows better performance than that of other two CSMA/CA protocols in low traffic load, while 4-WH CSMA/CA superior to others in high traffic load. In the case of fading, shadowing and power capture model, the performance of 4-WH CSMA/CA is always better than that of the other two protocols. Moreover, we note that the performance of CSMA/CA in the fading channel model is worse than that in the error free channel model when the traffic is low. Finally, we note that 4-WH CSMA/CA protocol is more appropriate than Basic CSMA/CA or SW ARQ CSMA/CA in practical wireless communication environments.

## VI. Conclusions

In this paper, we have analyzed the performance of CSMA/CA protocols with power capture, operating on a channel impaired by Rayleigh fading, lognormal shadowing and near-far effect. we have considered three types of CSMA/CA protocols, including Basic, SW ARQ and 4-WH CSMA/CA, and have analyzed their throughput and packet delay.

To analyze the performance of CSMA/CA, we have considered capture probability in fading and shadowing channels. We have found that capture probability is converged to a finite limit as the number of colliding terminals is increased. Then, we have developed a new analytical approximation for the performance of CSMA/CA protocols with Rayleigh fade, lognormal shadowing and power capture effect. From the analytical results, we have found that the throughput of CSMA/CA protocol is not decreased as the increase of the number of terminals and the offered load. We have also found that the performance of CSMA/CA is enhanced as the increase of transmission probability  $p$  and is sensitive the capture ratio  $z$ . Extensive numerical results have been presented to show that 4-WH CSMA/CA protocol is a more attractive protocols than the other two types of CSMA/CA in practical wireless communication environments.

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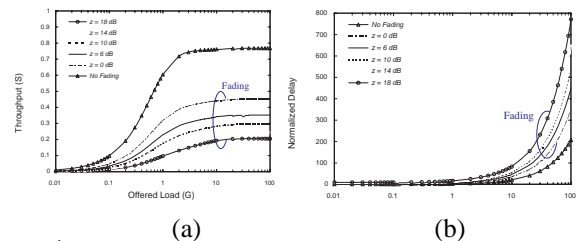


Figure 1: Throughput and packet delay vs. offered load of Basic CSMA/CA for varying of the capture ratio ( $a = 0.01, p = 0.03, f = 0.06, M = 5, Y = 0.06, \delta = 0.06, \beta = 0.03, \xi = 4$ ) (a) Throughput vs. offered load (b) Normalized packet delay vs. offered load

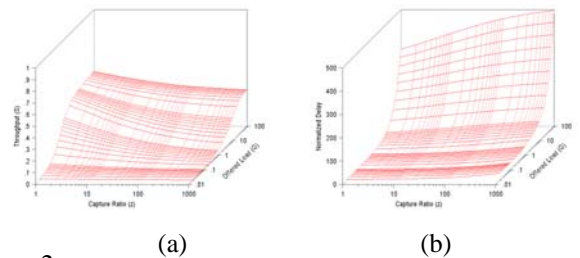


Figure 2: Throughput and packet delay vs. capture ratio of CSMA/CA with acknowledgement for varying of the offered load ( $a = 0.01, p = 0.03, f = 0.06, \delta = 0.06, \beta = 0.03, M = 20, Y = 0.06, \sigma_s = 6dB, \xi = 4$ ) (a) Throughput vs.  $z$  (b) Normalized packet delay vs.  $z$ .

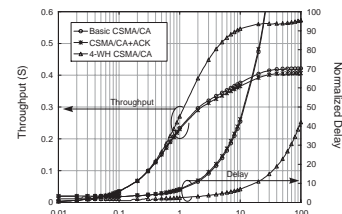


Figure 3: Performance comparison of three types of CSMA/CA protocols for error channel model and fading channel model. ( $a = 0.01, p = 0.03, f = 0.06, \delta = 0.06, \beta = 0.03, \theta = 0.06, \gamma = 0.1, M = 30, Y = 0.06, \sigma_s = 6dB, \xi = 4$ ) (a) Throughput comparison (b) Normalized packet delay comparison.