The Optimization of the Collision Resolution Algorithm for Broadband Wireless Access Network

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***Abstract* — In this paper, we analyze the performance of the binary exponential backoff algorithm in the broadband wireless access (BWA) network. Especially, we focus on the initial backoff window for improving the performance of the BWA medium access control (MAC) protocol. To analyze the optimal initial backoff window size, we define the cost function using the system performance and calculate the optimal initial backoff window according to the number of users and the traffic characteristics.**

***Keywords* — IEEE 802.16 MAC protocol, binary exponential backoff, optimal initial backoff window**

1. Introduction

IEEE 802.16 standard assumes a point-to-multipoint topology, with a controlling base station (BS) that connects subscriber station (SS) to various public networks linked to the BS. The access mode for the uplink channel has been considered as time division multiple access (TDMA) and orthogonal frequency multiple access (OFDMA), but we have focused on TDMA in this paper. Each uplink channel is composed of fixed-size minislots. The uplink channel is divided into concatenated frames, and each frame is composed of the contention period (CP), the data transmission period (DTP), and the initial ranging period (IRP) allocated by a bandwidth allocation MAP message. In IEEE 802.16 MAC protocol, the bandwidth request message is transmitted to obtain the data transmission bandwidth in the CP.

IEEE 802.16 standard supports the service flows to guarantee the quality of service (QoS). The four service flows are defined as follows.

- UGS is generated to support real-times service flows that generate fixed-size data packets on a periodic basis, such as Voice over IP.
- rt-PS is designed to support real-time service flows that generate variable-size data packets on a periodic basis, such as MPEG video.
- nrt-PS is designed to support non-real-time service flows that require variable-size data grants on a regular basis, such as high bandwidth FTP.
- BE : Neither periodic polls nor periodic data grants will be sent by the BS unless they are needed to satisfy the minimum reserved bandwidth for that service.

Since a collision can be occurred between request messages, the truncated binary exponential backoff algorithm is applied to the IEEE 802.16 MAC protocol. In the binary exponential backoff algorithm, if collision is occurred, the collided request message is retransmitted within the backoff window which increases by a multiple of two. The initial backoff window and maximum backoff window are controlled by the BS through the uplink channel descriptor (UCD) message. And BS knows the number of users through the processes of ranging and registration. Since the transmission and collision probability of the request message depend on the number of users, the system performance can be improved if the BS allocates the optimal initial backoff window to the SS according to the number of users [1]-[3].

In this paper, we analyze the performance of the IEEE 802.16 MAC protocol, we particularly concentrate on the performance of the system according to the initial contention window. We define the cost function using the system performance to analyze the optimal initial backoff window according to the traffic characteristic and the number of users.

For the related works, Bianchi and Kwak analyzed the performance of the binary exponential backoff algorithm for the IEEE 802.11[4], [5]. Bianchi and Kwak provided the analytic model for the performance of IEEE 802.11, and analyzed the initial backoff window and the maximum retry according to the number of users. However, they didn’t find the optimal initial backoff window according to the number of users and the traffic characteristic. Thus, we analytically model the optimal initial backoff window according to the number of users and traffic characteristic.

2. Analysis of Binary Exponential Backoff Algorithm in BWA MAC Protocol

For the simple analysis, we assume the uplink frame structure in Fig. 1 and the communication environment is as follows.

- The propagation delay is neglected.

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- The packet transmission error is not considered.
- The BS is able to detect the collision in the uplink channel.
- The system is regarded as the saturation condition.

For the binary exponential backoff, there are backoff counters correspond to the Transmission Opportunity. The Transmission Opportunity composed of minislots is the transmission unit of the request message. We define the system parameters based on the IEEE 802.16 standard in Table I[3]. In this paper, we define the Transmission Opportunity is equal to a slot made of eight minislots.

Table 1. System Parameters

System Parameter	Value
Transmission Opportunity	8 minislots
Modulation	QPSK
Data Rate	32 *Mbits/s*
The Size of a minislot	4 μs
The Size of CP	32 minislots

The stations retransmit the request message after waiting for the backoff time that is randomly selected within the backoff window whose range is from 0 to W_i, where W_i is a backoff window size for the i_{th} backlogged station. The backoff window increases by

$$W_i = r^i W_0 \quad (0 \le i \le M \text{ ,} \quad W_0 \le W_i \le W_{\max}) \tag{1}$$

where, W_0 is the initial backoff window, i is the number of retransmission, M is the maximum retry, and r is the backoff factor.

The throughput (S) is defined as the ratio of the number of slots successfully transmitted request messages and the total number of slots in the CP. Thus, the throughput is able to be defined as

$$S = \binom{N}{1} p_t (1-p_t)^{(N-1)} \tag{2}$$

where p_t is a transmission probability of a station for a slot and N is the number of users. We refer to [5] for the value of p_t according to the number of users.

Since D_i is the backoff counter randomly selected by an i_{th} retransmitted station, the waiting time of the i_{th} retransmitted station is D_i+1 (slots). The expectation of the waiting time is given by [4]

$$E\left[D_i + 1\right] = \frac{W_i - 1}{2} + 1 = \frac{W_i + 1}{2}. \tag{3}$$

N_R is defined as the number of retransmission. Noting that N_R and D_i, i=0,1,…., N_R are both random variables. The packet drop probability (p_d) is equal to p_c^{M+1}. Thus, the expectation of retransmission is driven as

$$\begin{aligned} n_R &= E\left[N_R \mid \text{ no drop}\right] = \sum_{n=0}^{M} n \frac{(1-p_c)\, p_c{}^n}{1-p_d} \\ &= \frac{p_c}{1-p_c} - \frac{(M+1)\, p_c{}^{M+1}}{1-p_c{}^{M+1}}. \end{aligned} \tag{4}$$

If a request message is collided, the request message will be retransmitted until n_R or dropped. For the retransmission of the collided request message, the request message has to be delayed by $n_R \times F$ (slots). ($\lfloor (D_i+1)/CP \rfloor$ and $\lfloor D_R / CP \rfloor$ are both the number of frame that has to wait for transmitting the request message. $\lfloor * \rfloor$ indicates the maximum integer smaller than the number ($*$).) The request message also waits for the remained backoff time in the last frame. ($D_i + 1 - \lfloor (D_i+1)/CP \rfloor \times CP$ is the remained backoff time after $\lfloor (D_i+1)/CP \rfloor$ frames.) Thus, the expectation of delay is defined as

$$\begin{aligned} \bar{D} = E\Bigg[\Bigg\{ & \sum_{i=0}^{N_R-1} \left(\left\lfloor \frac{D_i+1}{CP} \right\rfloor \times F + D_i + 1 - \left\lfloor \frac{D_i+1}{CP} \right\rfloor \times CP \right) \\ & + \left(\left\lfloor \frac{D_{N_R}}{CP} \right\rfloor \times F + D_{N_R} - \left\lfloor \frac{D_{N_R}}{CP} \right\rfloor \times CP \right) \Bigg\} \Bigg| \text{ no drop} \Bigg]. \end{aligned} \tag{5}$$

Therefore, the average delay for the successful transmission is given by

$$\begin{aligned} \bar{D} = & \frac{W_0}{2(1-r)} \times \left(1 - \frac{(1-p_c) r}{1-p_c{}^{M+1}} \sum_{n=0}^{M} (r p_c)^n \right) \times \frac{F}{CP} \\ & + \frac{1}{2} \times \left(\frac{p_c}{1-p_c} - \frac{(M+1)\, p_c{}^{M+1}}{1-p_c{}^{M+1}} + 1 \right) \times \left(\frac{F}{CP} - F + CP \right) \\ & - \frac{F}{CP}. \end{aligned} \tag{6}$$

The derivation of (6) can be found in Appendix. For the binary exponential backoff algorithm, r is equal to 2, the average delay is given by

$$\begin{aligned} \bar{D} = & \frac{W_0}{2} \times \left(\frac{2(1-p_c)}{1-p_c{}^{M+1}} \sum_{n=0}^{M} (2 p_c)^n - 1 \right) \times \frac{F}{CP} \\ & + \frac{1}{2} \times \left(\frac{p_c}{1-p_c} - \frac{(M+1)\, p_c{}^{M+1}}{1-p_c{}^{M+1}} + 1 \right) \times \left(\frac{F}{CP} - F + CP \right) \\ & - \frac{F}{CP}. \end{aligned} \tag{7}$$

3. Cost Function Modeling and Optimal Initial Backoff Window Analysis

For the cost function(CF), we normalize the throughput, delay, and packet drop probability. First of all, the inverse throughput (S') is defined as

$$S'(N, W_0) = \frac{1}{S(N, W_0)}. \tag{8}$$

Thus, the normalized inverse throughput is given by

$$Nor\_inv\_throughput(N, W_0) = \frac{S'(N, W_0) - S'_{\min}(N, W_0)}{S'_{\max}(N, W_0) - S'_{\min}(N, W_0)}. \tag{9}$$

And the normalized delay is

$$Nor\_delay(N, W_0) = \frac{D(N, W_0) - D_{\min}(N, W_0)}{D_{\max}(N, W_0) - D_{\min}(N, W_0)}. \tag{10}$$

The normalized packet drop probability is

$$Nor\_prob\_drop(N,W_0)=\frac{P_d(N,W_0)-P_{d\min}(N,W_0)}{P_{d\max}(N,W_0)-P_{d\min}(N,W_0)}. \tag{11}$$

Using (9), (10), and (11), we define the cost function as

$$\begin{aligned} CF(N,W_0) &= WF_1\times Nor\_inv\_throughput(N,W_0) \\ &+WF_2\times Nor\_delay(N,W_0) \\ &+WF_3\times Nor\_prob\_drop(N,W_0) \end{aligned} \tag{12}$$

where WF_i is the weighting factor for the throughput, delay and packet drop probability (Respectively i = 1, 2, 3).

4. Numerical Results

As the mathematical analysis mentioned above, we analyze the performance of the BWA MAC protocol in terms of throughput, delay and packet drop probability. And we investigate the optimal initial backoff window size according to the traffic characteristic and then number of users.

Fig. 2 shows the throughput when the maximum retry is 6, and CP is equal to the 32 minislots. The throughput increases by a certain point, and then decreases as the number of users increases. And we know the maximum throughput depends on the size of the initial backoff window. However, we approximately consider that the throughput does not depend on the initial backoff window, because the difference of the throughput by the size of the initial backoff window is very small.

In Fig. 3, the curves indicate the delay according to the number of users and the size of the initial backoff window. The delay increases as both the number of users and the initial backoff window increase, because of the number of retransmission and fragmented frames. Note that the difference of the delay between W_0=4 and W_0=32 is about 350 slots which is very large value for the delay. Therefore, we have to consider the delay as the important performance factor for the delay sensitive service such as UGS or rt-PS.

Fig. 4 presents the packet drop probability versus the number of users and initial backoff window. The packet drop probability is affected by the collision probability of the request messages. Thus, the packet drop probability increases as the number of users increases or the initial backoff window decreases. When the number of users is equal to 100, the difference of the packet drop probability according to the initial backoff window is very large. Hence, we also consider the packet drop probability as the performance factor for the service which is critical to the packet error rate such as nrt-PS or BE.

Considering the performance analytic results, we know that the delay and packet drop probability have to be considered to improve the system performance. Therefore, we define the cost function according to the weighting factor. We assume the proportion of the weighting factor by the traffic characteristics such as 0.33:0.33:0.33, 0.1:0.7:0.2, and 0.1:0.2:0.7.

1) When the weighting factors are same. (WF_1=0.33, WF_2=0.33, WF_3=0.33)

In Fig. 5, the curves indicate the cost function according to the number of users. The system presents a good performance when the cost function is small. We find the optimal initial backoff window according to the number of users in the Fig. 5, and propose the optimal contention window in Table II.

Table 2. Optimal Initial Backoff Window According to The Number of Users

The number of users	Optimal Initial Backoff Window
2~17	$W_0 = 4$
17~85	$W_0 = 8$
85~100	$W_0 = 16$

2) The system is sensitive to the delay (WF_1=0.1, WF_2=0.7, WF_3=0.2)

In Fig. 2, the throughput does almost not change according to the initial backoff window. Thus, we determine the weighting factor of the throughput as 0.1. Fig. 6 presents the cost function according to the number of users and the initial backoff window. As shown in Fig. 6, we had better reduce the initial backoff window if we consider the delay sensitive system. Based on the analytic result, we propose the optimal initial backoff window is 4.

3) The system is affected by the packet drop probability (WF_1=0.1, WF_2=0.2, WF_3=0.7)

In Fig. 7, the curves present the cost function when we choose the weighting factor as the system which is critical to the packet drop probability. In this system, if we intuitively consider we had better increase the initial backoff window, but the optimal size of the initial backoff window is almost 16 according to the analytic result based on Fig 7. We particularly propose the optimal initial backoff window in Table III.

Table 3. Optimal Backoff Window According to The Number of Users

The number of users	Optimal Initial Backoff Window
2~6	$W_0 = 4$
6~16	$W_0 = 8$
16~70	$W_0 = 16$
70~100	$W_0 = 32$

5. Conclusion

The truncated binary exponential backoff algorithm is applied to the collision resolution mechanism in IEEE 802.16 MAC protocol. We analyzed the binary exponential backoff algorithm applied with the framed slotted aloha, and designed the cost function with weighting factors to compare the system performance between the various systems, and then proposed the optimal initial backoff window considering the system characteristics. The proposed mechanism for optimal initial backoff window can be applied to the collision resolution mechanism, and MAC protocol in IEEE 802.11, IEEE 802.16e and DOCSIS. For the future works we are working on the optimization of the maximum retry, and the development of the improved collision resolution mechanism.

APPENDIX

For the derivation of the delay, the following process is needed.

$$\begin{aligned}\bar{D} = E\Bigg[\Bigg\{\sum_{i=0}^{N_R-1}\left(\left\lfloor\frac{D_i+1}{CP}\right\rfloor\times(F-CP)+D_i+1\right)\\ +\left(\left\lfloor\frac{D_{N_R}}{CP}\right\rfloor\times(F-CP)+D_{N_R}\right)\Bigg\}\mid \text{no drop}\Bigg].\end{aligned} \tag{13}$$

Where

$$\left\lfloor\frac{D_i+1}{CP}\right\rfloor=\frac{D_i+1-y}{CP} \tag{14}$$

$0 \le y < CP$, y is a uniformly distributed random variable.

$$\left\lfloor\frac{D_{N_R}}{CP}\right\rfloor=\frac{D_{N_R}-y'}{CP} \tag{15}$$

$0 \le y' < CP$, y' is a uniformly distributed random variable.

Substituting (14) and (15) into (13), the delay is given as

$$\begin{aligned}\bar{D} = E\Bigg[\Bigg\{\sum_{i=0}^{N_R-1}\left(\frac{D_i+1}{CP}\times F-\frac{y}{CP}(F-CP)\right)\\ +\left(\frac{D_{N_R}}{CP}\times F-\frac{y'}{CP}(F-CP)\right)\Bigg\}\mid \text{no drop}\Bigg].\end{aligned} \tag{16}$$

Since y and y' are a uniformly distributed random variable.

$E[y \mid \text{no drop}]=E[y]=\frac{CP}{2},\ E[y' \mid \text{no drop}]=E[y']=\frac{CP}{2}.$

Thus, the delay can be driven as

$$\begin{aligned}\bar{D} &= E\Bigg[\Bigg\{\sum_{i=0}^{N_R-1}\left(\frac{D_i+1}{CP}\times F-\frac{1}{2}(F-CP)\right)\\ &\quad +\left(\frac{D_{N_R}}{CP}\times F-\frac{1}{2}(F-CP)\right)\Bigg\}\mid \text{no drop}\Bigg]\\ &= E_{N_R}\Bigg[\sum_{i=0}^{N_R}\frac{W_i+1}{2}\times\frac{F}{CP}\\ &\quad -\frac{(N_R+1)(F-CP)}{2}-\frac{F}{CP}\mid \text{no drop}\Bigg]\\ &= E_{N_R}\Bigg[\frac{W_0}{2}\left(\frac{1-r^{N_R+1}}{1-r}\right)\times\frac{F}{CP}\\ &\quad +\frac{(N_R+1)}{2}\times\left(\frac{F}{CP}-F+CP\right)\mid \text{no drop}\Bigg]-\frac{F}{CP}.\end{aligned} \tag{17}$$

Since the equation (16) is the expectation of $N_{R,}$

$$\begin{aligned}E\left[r^{N_R+1}\mid \text{no drop}\right]&=\sum_{n=0}^{M}r^{n+1}\frac{(1-p_c)p_c{}^n}{1-p_c{}^{M+1}}\\ &=\frac{(1-p_c)r}{1-p_c{}^{M+1}}\sum_{n=0}^{M}(rp_c)^n.\end{aligned} \tag{18}$$

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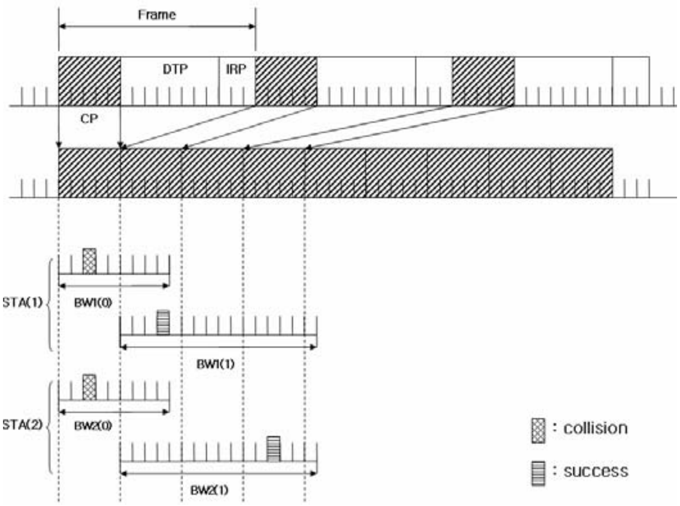

Figure 1. Uplink frame Structure and Backoff Window for Uplink Channnel

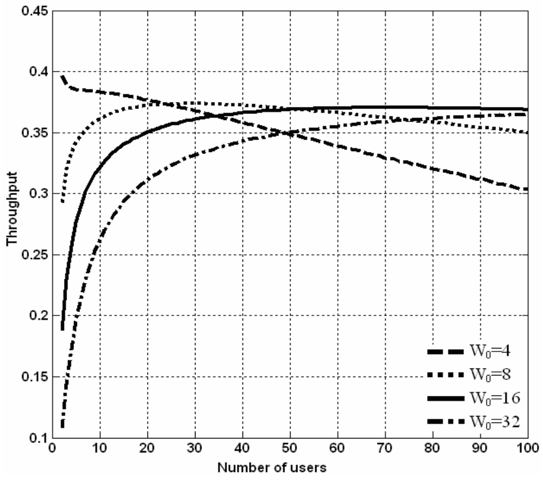

Figure 2. Throughput according to the number of users (*M*=6)

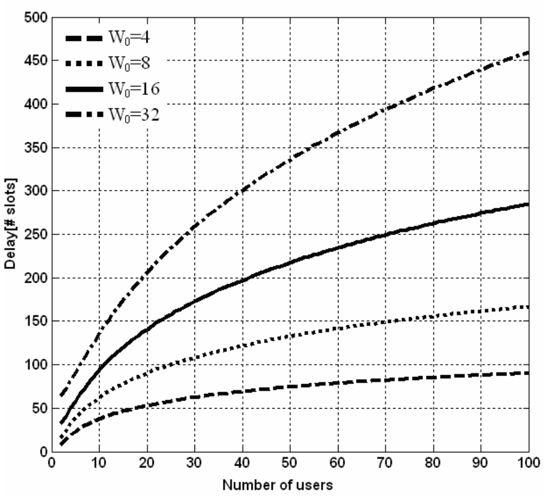

Figure 3. Delay according to the number of users (*M*=6)

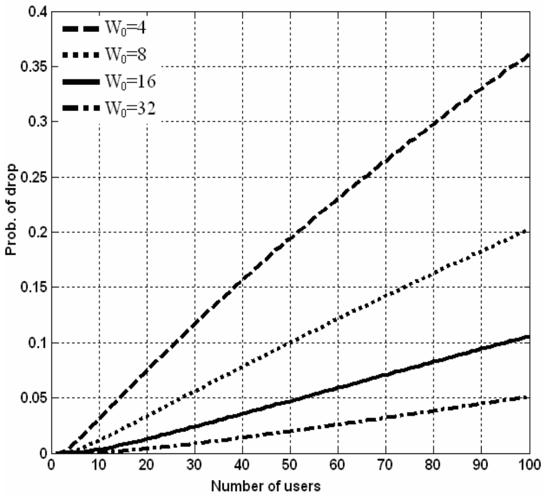

Figure 4. Packet drop probability according to the number of users (*M*=6)

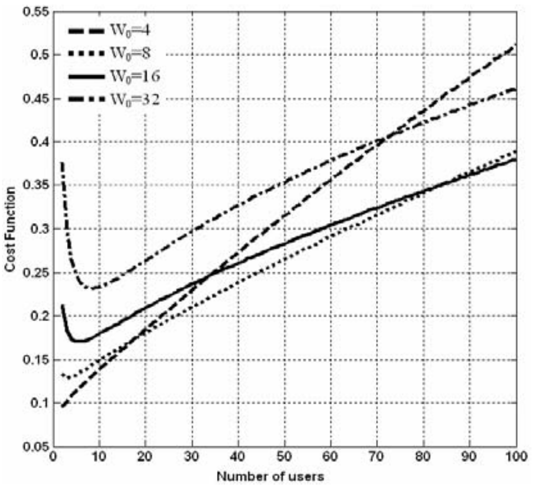

Figure 5. Cost function according to the number of users (*WF*$_1$=0.33, *WF*$_2$=0.33, *WF*$_3$=0.33)

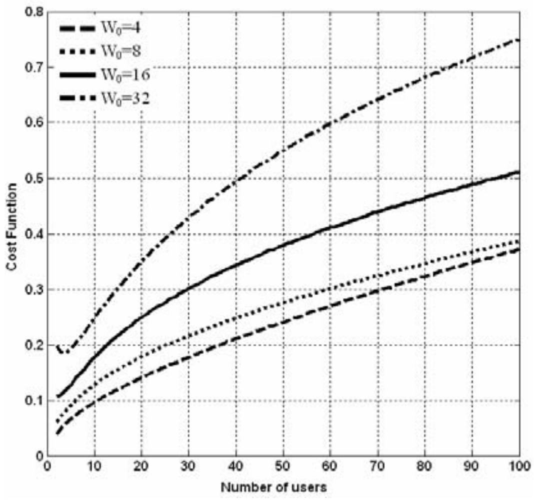

Figure 6. Cost function according to the number of users (*WF*$_1$=0.1, *WF*$_2$=0.7, *WF*$_3$=0.2)

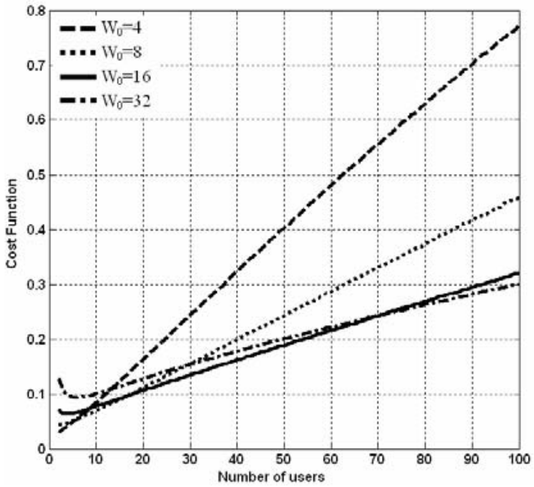

Figure 7. Cost function according to the number of users (*WF*$_1$=0.1, *WF*$_2$=0.2, *WF*$_3$=0.7)